

# Hooke's Law

**Concepts**

- Graph scatter plots
- Analyze and graph linear functions
- Calculate and model slope

**Materials**

- TI-84 Plus
- CBR 2™ (Calculator-Based Ranger 2™)
- Vernier® EasyData™ Application
- Small spring cut in half (a Slinky® works well)
- Empty 4oz plastic/paper cup (flat bottom)
- Two large paper clips or string
- Meter stick or other long stick
- Second meter/yard stick or ruler
- 30 Candies (M&Ms® and almonds work well)

**Overview**

In this activity, Hooke's Law is explained in its simplest form for beginning algebra students.

**Introduction**

Hooke's Law states that the stretch on a spring is directly proportional to the force applied to the end of the spring. The force is the weight of an increasing amount of candy added to a cup. This force is applied to the end of a spring.

The data will be collected using the CBR 2™ and the EasyData™ APP. This experiment will produce linear behavior. You will analyze the data, determine an equation of the line of best fit, and interpret the meaning of the slope. The coordinates of the graph are the distance from the cup to the floor vs. the number of candies. You will interpret the values used in your model and apply the properties of this linear model to predict future behavior/events.

## Setup

Briefly demonstrate and discuss the data collection procedure with the class. Have students collect the data in groups of 3 or 4. Each group should do the following:

1. Place a meter stick or other long stick on the back of two chairs as shown in the diagram on the previous page.
2. Hang the spring from the stick.
  - A plastic Slinky® cut in half works well.
3. Poke holes in the cup and put string or paper clips through the holes to form a handle.
  - You can also use an empty cream cheese or margarine container. The container you choose needs to have a flat bottom so the CBR 2™ can take a reading of it.
  - You can save valuable class time by having the handles already attached before the experiment.
  - Hang the cup/container on the spring with the string/paper clip handle.
  - Make sure it is at least 18–24 inches off the floor when empty.
4. Before starting, have students measure the distance, in inches, from the top of the stick to the floor and record this on their worksheet.
5. Position the CBR 2™ on the floor directly under the empty cup/container so you can measure the distance from the floor to the bottom of the container.
  - Link the CBR 2™ directly to the TI-84 Plus.
  - You can use either the I/O unit-to-unit cable or the mini-USB cable.
6. The EasyData™ APP will launch automatically if you are using the mini-USB cable.
  - If you are using the I/O unit-to-unit cable, you will need to press **[APPS]**, scroll down to highlight the EasyData™ APP, and then press **[ENTER]** in order to launch the APP.

## Data Collection

- Press  $\boxed{Y=}$  to access the **File** menu, and select **1:New** by pressing  $\boxed{1}$ .
  - Or since **1:New** is highlighted, you can just press  $\boxed{\text{ENTER}}$ .
  - This resets the program and clears out old data (Figure 1).
- The default unit of measurement on the EasyData™ APP is meters. You will do this activity in feet.
  - To change the units of measurement, press  $\boxed{\text{WINDOW}}$  on the top row of the calculator to select the **Setup** menu soft key.
  - From the **Setup** menu, choose **1:Dist** by pressing  $\boxed{1}$  or  $\boxed{\text{ENTER}}$  since **1** is highlighted (Figure 2).
- From the **Units** menu, select **2:(ft)** by pressing  $\boxed{2}$  or by pressing the down arrow key until the **2** is highlighted and pressing  $\boxed{\text{ENTER}}$  (Figure 3).
  - When you have confirmation that you will be using feet, select OK (Figure 4).
- You will be returned to the main screen of the EasyData™ APP.
  - From the **Setup** menu, select **6: Events with Entry**.
  - This will allow you to control when data is recorded by pressing a key on the calculator (Figure 5).
- This takes you back to the main screen.
  - Select **Start**.
  - You will hear the CBR 2™ clicking as the reading is displayed at the top of the screen of the calculator.
  - Select **Keep** to record this first data reading.
  - It is the distance from the bottom of the empty container to the CBR 2™ (Figure 6).



Figure 1



Figure 2

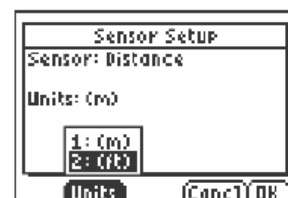


Figure 3



Figure 4

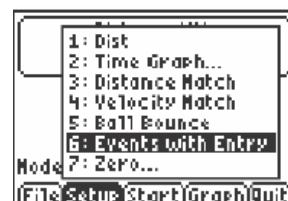


Figure 5

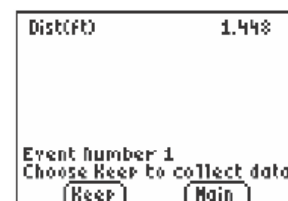


Figure 6

6. When the **Enter Value** screen appears, press **0**, for no candies (Figure 7).
7. Slowly add 3 candies, and try to minimize the swing of the container.
  - When the container is steady and the distance displayed at the top of the screen is stable, select **Keep** again.
  - This time enter 3 in the **Enter Value** screen to represent the 3 candies in the cup (Figure 8).
8. Continue for a total of 10 trials.
  - Each time the **Enter Value** screen appears, enter the value of 3 more candies than the previous trial.
  - With each recorded value, a new data point will be displayed on the graph along with the option to **Keep** or **Stop** the data collection (Figure 9).
9. When finished with the trials, select **Stop**.
  - A graph of your data points will be displayed.
  - Use the right and left arrow keys to view the values of the points (Figure 10).
10. To confirm a description of the plots, press **WINDOW** to select the **Plots** soft key.
  - This is a good time to review the vocabulary with your students.
  - Identify the **Events** as the independent variable and the Distances as the dependent variable (Figure 11).
11. Select **Anlyz**, and choose **2:Linear Fit** from the menu (Figure 12).

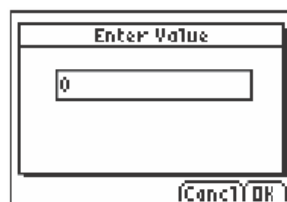


Figure 7

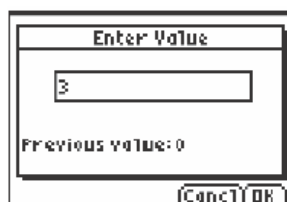


Figure 8

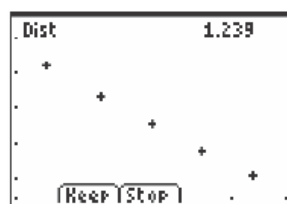


Figure 9

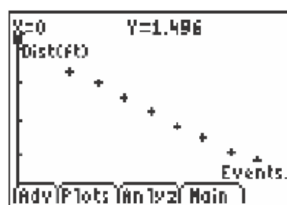


Figure 10

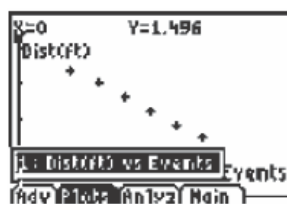


Figure 11



Figure 12

12. When **2:Linear Fit** is chosen, the calculator will display an equation for the line of best fit.
  - Select OK (Figure 13).
13. You will see the line of best fit being drawn on the screen with the data points.
  - You can still use the right and left arrow keys to scroll through the data points.
  - Use either the up or down arrow key to trace along the line rather than on the individual points (Figure 14).
14. Select **Main**, and then **Quit** on the next screen.
  - The confirmation screen will be displayed telling you the lists where your data is stored.
  - Select OK to exit the APP (Figure 15).

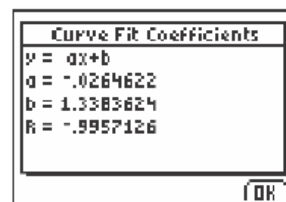


Figure 13

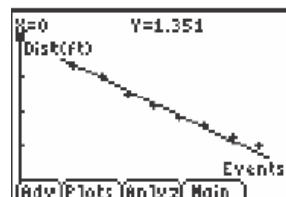


Figure 14

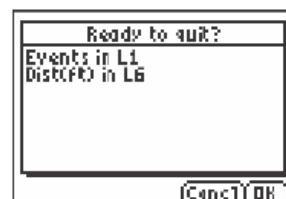


Figure 15

## Data Analysis

1. Rather than having the calculator do all the work of finding the regression equation, consider having the students use the definition of slope and their understanding of linear equations to write their own equation for the line of best fit.
  - An additional step would be to find the equation using both methods, and then compare the answers.
  - The following directions use a combination of the knowledge of formulas and calculator computations to graph an equation of a line.
  - The students need to know the formula for deriving the slope when given the coordinates of two points and how to use the calculator for quicker and more accurate computations.
  - This activity is meant to build the students' understanding of slope while showing them some lesser used features of the calculator.
2. The exit screen of the APP verified that the data values you recorded are in lists **L1** and **L6**.
  - Under the [STAT PLOT] menu, the **Plot1** is turned on with the window set to display all the points collected.
  - **Y1** is turned off, but the regression equation found by the APP is still displayed there.

- You can tell when the **Y1** is turned off because the equals sign is no longer highlighted.
  - This indicates that the equation's graph will not be displayed in the **GRAPH** window (Figure 16).
3. If you have already completed some of the previous lessons, this may be a good time for participants to practice on their own and/or in groups.
- You could circulate the room and help where needed as students find their own line of best fit and compare it to the regression equation found by the calculator.
  - You can decide how to proceed from here based on your students' knowledge and what your goals are for this lesson.
  - Have the students link **L1**, **L6**, and **Y1**. This will allow each student to do his/her own data analysis.

**NOTE:** The following is a slightly different method of finding the slope of the regression equation. This method uses some of the features under the list menu. During this procedure, the student's knowledge of the definition of slope will be reinforced even though they will use technology to perform the otherwise time-consuming calculations. You may find it easiest to have the students work in groups to collect and link the data. Then they can come back together as a class while you direct them. They may need your direction because the features used in the next steps could be new to them.

4. It will be helpful to have **L1** and **L6** next to each other in the display.
- To accomplish this, press **[STAT]** **[ENTER]** to access the Stat List Editor; and position the cursor so the name **L2** is highlighted.
  - Press **[2nd]** **[DEL]** to access the **[INS]** (insert) command (Figure 17).
5. A blank column will be inserted to the right of **L1**.
- Press **[2nd]** **[6]** to have **L6** appear at the bottom of the screen (Figure 18).
  - Press **[ENTER]** to see **L6** fill in the blank column (Figure 19).

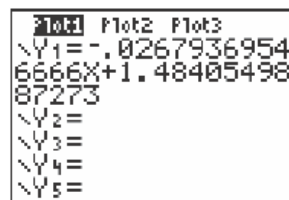


Figure 16

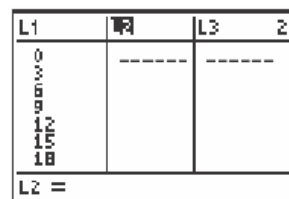


Figure 17

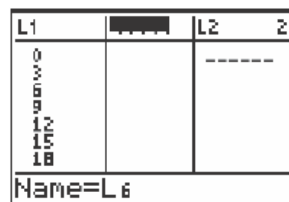


Figure 18

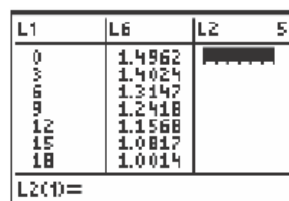


Figure 19

6. We already know the differences in the **X**-values of any two consecutive points is three because the number of candies increased by three for each trial.
  - The goal for this next set of calculator commands is to use the calculator to determine the differences in the **Y**-values of the data points collected in the activity, and then use that difference to find the slope.
7. Give **L2** a command that will calculate the difference between two consecutive values in **L6** and put that difference in **L2**.
  - To do this, use the up arrow key to highlight **L2**.
  - Press **[2nd]** **[STAT]** to access the **[LIST]** menu, and arrow over to the **OPS** menu.
  - Select **7: ΔList** (Figure 20).
8. You will see this command displayed at the bottom of the Stat List Editor window.
  - Press **[2nd]** **[6]** to type in **L6**, and close the parenthesis (Figure 21).
9. This will give the differences in the **Y**-values of your data points that were stored in **L6**.
  - Press **[ENTER]** to see **L2** filled in with these differences (Figure 22).
10. Because the change in the **X**-values is always three, you can divide each difference by three to find the slope for the individual segments between any two consecutive points listed in **L6**.
  - Position the cursor to highlight **L3**, and define it to be **L2/3** (Figure 23).
11. Press **[ENTER]** to see **L3** filled in with these slopes (Figure 24).
  - Discuss with your students why these differences, although very close, are not exactly the same.
  - This is a good time to discuss the difference between theoretical events and actual data collection.
  - Point out how human error affects data collection, and that the CBR 2™ helps to cut down on that type of error.
  - Discuss the factors that could contribute to human error if the measurements were taken by hand with a yard or meter stick.

```

NAMES OPS MATH
1:SortA(
2:SortD(
3:dim(
4:Fill(
5:seq(
6:cumSum(
7:ΔList(

```

Figure 20

L1	L6	NAME	3
0	1.4962		
3	1.4024		
6	1.3147		
9	1.2418		
12	1.1568		
15	1.0817		
18	1.0014		

L2=ΔList(L6)

Figure 21

L1	L6	L2	3
0	1.4962	-.0938	
3	1.4024	-.0877	
6	1.3147	-.0729	
9	1.2418	-.085	
12	1.1568	-.0751	
15	1.0817	-.0802	
18	1.0014	-.07	

L2(-)=-.093872688...

Figure 22

L6	L2	NAME	4
1.4962	-.0938		
1.4024	-.0877		
1.3147	-.0729		
1.2418	-.085		
1.1568	-.0751		
1.0817	-.0802		
1.0014	-.0797		

L3=L2/3

Figure 23

L6	L2	L3	4
1.4962	-.0938	-.0312	
1.4024	-.0877	-.0292	
1.3147	-.0729	-.0243	
1.2418	-.085	-.0283	
1.1568	-.0751	-.025	
1.0817	-.0802	-.0268	
1.0014	-.0797	-.0266	

L3(-)=-.031266666...

Figure 24

- Examine how much variation there is between any two entries in **L3**.
  - Is .005 of a foot a significant error?
12. Guide the students to the understanding that the average of **L3** would serve as a close approximation for the slope of the line of best fit for all the points in the plot.
- Press **2nd** **MODE** to access **[QUIT]**, and return to the home screen.
  - Have the calculator find this average, and store it in **M**.
  - To do this, press **2nd** **[LIST]** and arrow over until **MATH** is highlighted.
  - Select **3:mean(** from the list (Figure 25).
13. Press **2nd** **[3]** to access **L3**.
- Complete the command by pressing **)** **STO►** **[ALPHA]** **M** **[ENTER]** (Figure 26).
14. Next, identify the **Y**-intercept.
- Return to the graph, and trace to the first data point (Figure 27).
15. Press **2nd** **MODE** to **[QUIT]**, and return to the home screen.
- The calculator keeps the **X**- and **Y**-values from the last point you traced in its memory until you trace to a new point.
  - Store the **Y**-value from first data point in the **B** variable.
  - The keystrokes are as follows: **[ALPHA]** **Y** **STO►** **[ALPHA]** **B** **[ENTER]** (Figure 28).
16. Next, check to see if these values for **M** and **B** are a close fit to the data.
- Go to the **Y=** window, and press **[ALPHA]** **M** **[X,T,θ,n]** **+** **[ALPHA]** **B** to type in **MX+B** next to **Y2** (Figure 29).
17. Press **GRAPH** to see how closely this equation fits the points.
- In the example shown, it looks like a great fit.
  - Press **TRACE**, and use the left and right arrow keys to scroll through the data points (Figure 30).

Figure 25

Figure 26

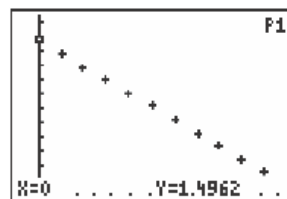


Figure 27

Figure 28

Figure 29

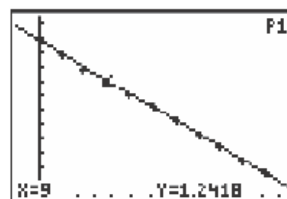


Figure 30



18. Next, press the up arrow.

- Your cursor will jump to the middle of the line and display the coordinates of the points on the line of the regression equation instead of the individual points from the lists.
- The upper left corner of the screen tells you the location of the equation you are tracing.
- In this example, it is tracing **Y2** (Figure 31).

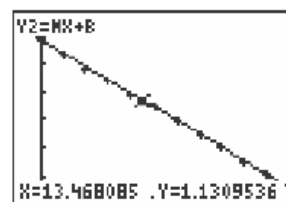


Figure 31

19. To compare how closely the regression equation you found matches the one the calculator found, graph them both at the same time.

- To distinguish between the two lines, use different graph styles.
- Go back to the  $Y=$  window, and turn on **Y1** by positioning the cursor on the equal sign and pressing **ENTER** to highlight it.
- Leave **Y1** with the default style, but use the left arrow key to highlight the slash icon in front of **Y2**.
- Repeatedly press **ENTER** until you see the symbol shown in Figure 32.
- The symbol looks like a ball with a line to its left.

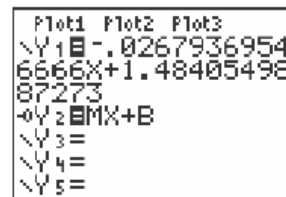


Figure 32

20. Press **GRAPH**.

- **Y1** is graphed using the default graph style.
- After **Y1** is completely graphed, you will see a small ball marking the trail as **Y2** is graphed.
- This feature makes it easy to see how closely your graph matches the graph found by the calculator (Figure 33).

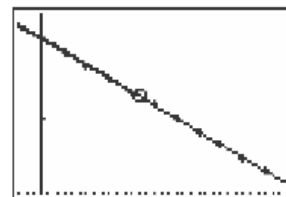


Figure 33

21. If you will be doing later activities, it is a good idea to save the data from **L1**, **L2**, and **L6** into named lists.

- Running the EasyData™ APP will overwrite any data that was saved in unnamed lists as will many other APPs and programs.
- Name **L1**, **HSEQ**, **L2**, **HDIF**, and **L6**, **HOOKS** (Figure 34).

HSEQ	HOOKS	HDIF	G
0	1.4962	-.0938	
3	1.4024	-.0877	
6	1.3147	-.0729	
9	1.2418	-.085	
12	1.1568	-.0751	
15	1.0817	-.0803	
18	1.0014	-.0797	
HDIF() = -.0938			

Figure 34

**NOTE:** If you run the EasyData™ APP again to collect another set of data, the previous data is overwritten. The EasyData™ APP will run with no probe connected if you want to use it to view your graph. A screen confirming that no interface is connected will be displayed. Select **None** to continue without a probe (Figure 35).

22. When the main screen is displayed, select **Graph** to view the plot from the last time data was collected (Figure 36).

## WORKSHEET ANSWERS

Answers will vary on problems not listed.

2. Number of candies
3. Distance from CBR 2™
4. Difference
5. Difference between 2 consecutive elements in **L6**.
6. **L2/3** calculates the slopes of each segment and the average is a close approximation for slope of the whole line.
7. Negative. As the number of candies increases, the distance decreases.
8. Less change...so less steep
11. Answers will vary, but they should match the slope in 9 or 10 when multiplied by 3.
13. The change in distance per change in candies.
15. a)  $y = m(10) + b$  b)  $12 = mx + b$



Figure 35

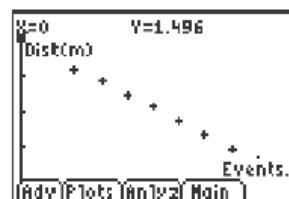


Figure 36

## Hooke's Law

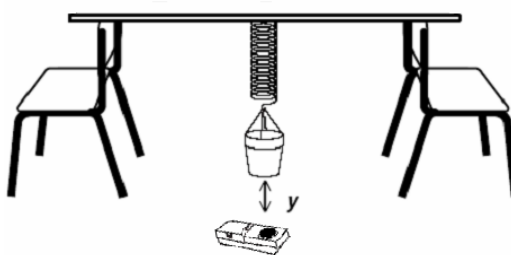
## Student Worksheet

Name \_\_\_\_\_

Class \_\_\_\_\_

**Overview**

In this activity, you will create a situation that produces linear behavior by observing a spring being stretched as you add candies to a cup attached to the spring. You will then apply the properties of a linear function to develop a model for your motion. Finally, you will interpret the values used in your model. Your teacher will outline the procedure for you. Use the results of the activity to fill in the blanks below.

**Setup**

Work in groups of 3 or 4 to do the following:

1. Place a meter stick or other long stick on the back of two chairs as shown in the diagram.
2. Hang the spring from the meter stick.
3. Hang the cup/container on the bottom of the spring.
  - Make sure that when the cup is empty, it is at least 18–24 inches off the floor.
4. Use a ruler or a second yardstick to measure the distance, in inches, from the top of the stick to the floor and record it here. \_\_\_\_\_.
  - This measurement will be used if you choose to do later activities.
5. Position the CBR 2™ on the floor directly under the empty cup/container so you can measure the distance from the floor to the bottom of the container.
  - Link the CBR 2 directly to the TI-84 Plus.
  - You can use either the I/O unit-to-unit cable or the mini-USB cable.
6. The EasyData™ APP will launch automatically if you are using the mini-USB cable.
  - If you are using the I/O unit-to-unit cable, you will need to press **[APPS]**, scroll down to highlight the EasyData™ APP, and then press **[ENTER]** in order to launch the APP.
  - Follow the procedure outlined by your Teacher.
7. While still in the EasyData™ APP, select **Anlyz** and select **2:Linear Fit**.
  - Record that equation here. \_\_\_\_\_

8. Link **L1**, **L6**, and **Y1** into each student's calculator.
9. Set up a scatterplot in **Plot1** with **L1** and **L6**, and sketch the graph of your data points in Figure 1.

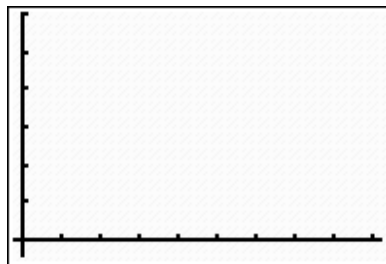


Figure 1

10. Before continuing, name three new lists in which to store and protect your data for use later.

- Store **L1** in a list named **HSEQ**, **L2** in a list named **HDIF**, and **L6** in a list named **HOOKS**.

Number of candies, L1	Distance from the bottom of the cup to the floor/CBR 2, L6
0	
3	
6	
9	
12	
15	
18	
21	
24	
27	
30	

- For recording purposes, round all decimals to the nearest hundredth.
1. Fill in the chart with the numbers from your calculator.
    - Follow your teacher's directions to find your own regression equation.
  2. What physical property is represented along the **X**-axis of the graph? \_\_\_\_\_
  3. What physical property is represented along the **Y**-axis? \_\_\_\_\_
  4. What is the mathematical meaning of the symbol delta,  $\Delta$ ? \_\_\_\_\_

5. Fill **L2** by using the command  $\Delta\text{List}(\text{L6})$ .

- Describe the meaning of the values this formula will generate and put into **L2**. \_\_\_\_\_

6. Define **L3** to be **L2** divided by 3 and then calculate the average/mean of the numbers in **L3**. Explain why.

7. Consider your trend line for the distance to the floor vs. the number of candies. Is the slope positive or negative? \_\_\_\_\_ Why?
8. What kind of slope would you predict for a stiffer spring than the one you used?
9. What is the regression equation you found to fit the data?
10. What was the regression equation found by the calculator?
11. For every set of three objects you put in the container, the distance from the CBR 2 decreased by.
12. What is the mathematical definition of slope?
13. Describe, in your own words, the meaning of slope in relation to this activity.
14. Use the **TRACE** or **TABLE** buttons to find the following:
  - a. How far from the floor would the cup be if 10 candies were used?
  - b. How many candies are in the can if it is 12 inches from the floor?
  - c. How many objects would it take until the container hits the floor?
15. Write the equations that you used to answer questions 14 a–c.
  - a) \_\_\_\_\_
  - b) \_\_\_\_\_
  - c) \_\_\_\_\_